Optimal Betting Spreads

The tables of Optimal Betting Spreads, in Chapters 6-10 of *The Brh Systems Book*, contain all the information which a player requires to design Betting Schemes which fit their style of play and bankroll. In order to the tables, it is necessary to understand how optimal spreads work, and what the various terms mean.

**Definitions**

**Betting Unit:**
A betting *unit* is the basic unit of a betting spread.

**Betting Spread:**
A 1-\( M \) betting *spread* is a series of values for each value of the (running or true) betting count, such that the minimum value is either zero or one unit, and the maximum value is \( M \) units.

**Unit Expectation (ev):**
The lowercase *ev* denotes the unit gain per round, for the counting system with a given betting spread.

**Unit Standard Deviation (sd):**
The lowercase *sd* denotes the unit standard deviation per round for a given betting spread.

**Long Run Index (\( N0 \)):**
The *Long Run Index* \( N0 \), is equal to

\[
N0 = \frac{sd^2}{ev^2},
\]  

which is the number of rounds which need to be played such that the total expectation is equal to the total standard deviation. It has a wider significance however, in that it is proportional to the bankroll doubling time for either a fixed or proportional bettor. The label \( N0 \) comes from the fact that if the player is losing by one standard deviation below the expected win, the after \( N0 \) rounds they will be even. For a spread that incorporates Wonging or backcounting, \( N0 \) also includes rounds watched but not played. However, the method used to generate the data in the tables incorporates all remaining unplayed rounds until the end of the shoe. This does not affect the accuracy of the betting spreads, but this extra factor may need to be taken into account if calculating, win rate per hour, in conjunction with the percentage (%pl) of rounds actually played.
Unit Equivalent Kelly Bankroll (\(ekb\)):

The Unit Equivalent Kelly Bankroll \(ekb\) is equal to

\[
ekb = \frac{sd^2}{ev}.
\]  

(2)

This quantity plays a crucial role in proportional (Kelly) betting schemes, and ‘Risk of Ruin’ calculations for fixed betting schemes.

Optimal Betting Spread:

The Optimal 1-\(M\) betting spread for a given ‘Wonging’ or backcounting strategy, is the spread that minimizes \(N_0\). The time to ‘get into the long run’ is minimized, so that fewer rounds need to be played to overcome an initial downswing of standard deviation or ‘bad luck’. However, since it is proportional to the doubling time, minimizing \(N_0\) ensures the fastest rate of bankroll growth.

Betting Scheme:

A betting scheme is different from a betting spread. A betting scheme \(B\) is obtained when a betting spread is multiplied by a monetary unit bet \(B\). For example a 1-10 betting spread, with a $5 unit bet, becomes a $5 to $50 betting scheme. The unit may change, and so the scheme will change, but the spread remains the same.

Expectation (EV):

The uppercase expectation \(EV\) for the betting scheme is equal to

\[
EV = B \cdot ev,
\]

and is the monetary expectation per round for the betting scheme \(B\).

Standard Deviation (SD):

The uppercase standard deviation \(SD\) for the betting scheme is equal to

\[
SD = B \cdot sd,
\]

and is the monetary standard deviation per round for the betting scheme \(B\). Note the standard deviation for \(N\) rounds is \(SD \times \sqrt{N}\), so that the \(SD\) for 100 rounds is only 10 times that for one round.

Optimal Betting Scheme:

An Optimal Betting Scheme is a betting scheme that incorporates an optimal spread. Note that the ratio of \(EV/SD\) is equal to the ratio of \(ev/sd\), so that \(N_0\) is always the same provided the same betting spread is used, regardless of the unit \(B\).
Bankroll:
The Bankroll is the total amount of money a player intends to risk playing blackjack. If this amount is lost, then further funds will not be found to continue. If a player has another source of income and is able to top up the bankroll after a loss, then the original bankroll is not the true bankroll. The total bankroll should be whatever you feel is your maximum limit, so that if you lose this you will quit the game forever. It is important that you consider what your real bankroll is, even if you do not have the funds available right now. Otherwise you will be not get the return on your blackjack investment that you desire, given the funds you are prepared to risk. If you lose your current mini-bankroll, then all you have to do is wait until your funds build up again, and use the time wisely to analyze your playing style and do a lot of practice at home. With good risk management, you should never lose your entire bankroll unless your play is deficient or you are extremely unlucky, although more likely the former. If you do lose your entire bankroll, then perhaps you need to consider that blackjack is not the game for you.

Equivalent Kelly Bankroll (EKB):
The uppercase Equivalent Kelly Bankroll for betting scheme with unit bet $B$, is

$$ EKB = \frac{SD^2}{EV} = B \cdot ekb. $$

(5)

This quantity is used to determine the bet $B$ for a given bankroll as well as the risk of ruin ($ROR$).

Fixed Bettor:
A fixed bettor is a player who maintains a constant value of the betting unit $B$. Such is the case if a $5 to $50 scheme is always used.

Proportional Bettor:
A proportional or Kelly bettor is a player who varies the betting unit $B$, depending on the size of the bankroll. Mathematically, this method will produce the fastest growth in the long run, provided that $B$ is always a fixed fraction of the bankroll. If $B$ is too large, it can lead to overbetting, where despite a positive expectation, the bankroll swings are so violent that at some point you will always lose your bankroll. It can be shown that if $k>2$, the bankroll will at some time reach zero, despite having a positive expectation. If $B$ is too small, there are no such dire consequences, but the rate of growth will be smaller than for the optimal value. That said, it is usually prudent to maintain some safety margin known as the Kelly factor $k$, which is less than the optimal value $k=1$ (pure Kelly) to prevent overbetting, and keeps bankroll swings under control. As will be explained later, a Kelly factor less than one, also allows a fixed bettor to make a smooth transition to proportional betting once the bankroll growth has begun.

Risk of Ruin ($ROR$):
The Risk of Ruin $ROR$ is the probability that a player, despite using a given counting system to perfect accuracy, loses all their bankroll nevertheless. It can be defined in a variety of ways depending on whether play is goal oriented, for example to double the bankroll, or for no goal such that play continues indefinitely. The latter is the mathematically the simplest to formulate, and that the form that is used here. Generally, risk of ruin is only a consideration for fixed bettors, but proportional bettors need to remember that there are minimum betting limits, and that they may in fact become fixed bettors should their bankroll become sufficiently small.
Use of Optimal Betting Tables

The first consideration is to find the spread with the lowest possible value of $N_0$ you can get away with. This entails as much backcounting and Wonging (leaving the table if count falls below some value) as possible, combined with spreading to two hands at the appropriate count value. One immediately sees that the play all shoe games are to be avoided at all costs, since the $N_0$ values are extremely high. That means that even with perfect counting play, one could go for years without winning. Also for these spreads, the value of $ekb$ is also quite high, which means a large bankroll is required, for any significant gain.

The second is the ability to spread to two hands in positive expectation. The values in these tables assume that there are three other players at the table. When the running or true count is greater than zero, the player spreads to two hands, each with a bet equal to the value in the table. Note that for equivalently sized spreads, the fractional bet value is less than for the single hand case. This is due to something known as covariance, where both hands are subject to the outcome of the dealer’s hand. The reason for spreading to two hands in this way, is that it is a good method for lowering $N_0$, although in this case, $N_0$ must now be interpreted as a number of rounds, rather than number of hands. You may not necessarily win quicker this way, but you will lower the swings in bankroll, provided you size the bets correctly.

Fixed Bettors

This kind of player has to be concerned with risk of ruin. Using the Unit Equivalent Kelly Bankroll ($ekb$), it is possible to either determine the bankroll required for a given unit bet $B$, or alternatively the value of $B$ for a given bankroll and risk of ruin.

Example:

Consider the 1-8 spread, play two hands $UTC \geq 0$, play only $UTC(Brh-I)$ greater than -6 (roughly equivalent to Hi-Lo $TC > -1$), example game given in the table below.

<table>
<thead>
<tr>
<th>TC</th>
<th>Spread</th>
<th>Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>0.00</td>
<td>$0</td>
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<td>$35</td>
</tr>
<tr>
<td>6</td>
<td>8.00</td>
<td>$40</td>
</tr>
</tbody>
</table>

$ekb$ 633 $3165
$N_0$ 31208
(Hi-Lo) 35678
%Pl 72.66
The 'Hi-Lo' row represents the value of $N_0$ a player could expect if the player were using the well known Hi-Lo count, details of which may be found in 'Professional Blackjack' by Stanford Wong. Hi-Lo is considered to be a standard of sorts, as it is the simplest balanced ace-reckoned count. Using any of the Brh Systems, including the running count versions, will outperform Hi-Lo. It is only since we have been able to compute optimal betting spreads, that a truly fair method of system comparison has been possible. For a given $1-M$ optimal betting spread, the system that delivers the lowest value of $N_0$, can truly be said to be the more powerful system. The use of the optimal betting spread, or equivalently the same bankroll for betting scheme construction, ensures that each system has the same risk of ruin, and so $EKB / N_0$ becomes the net rate of return on investment for a fixed bettor.

Another problem when comparing systems relates to the backcounting Wonging method and the percentage of rounds played (%pl). There is a question of fairness regarding the ability of a system to predict betting advantage. Some say that all systems must use the same Wong point in terms of (dis)advantage, for example leave when each system predicts an advantage of -1%. I personally have used the method where the percentage of rounds played for each has been matched. For true count systems there is usually little difference between the two, but there are differences between the two methods when comparing running count systems due to the non-linear advantage curve. In the tables in the Systems Book, I have attempted to match the percentages, when comparing with Hi-Lo, but in some cases it was either not possible or required an awkward number as a running count Wong point. The percentage reported in the table is for the particular Brh system, not Hi-Lo. This explains some discrepancies, particularly with running Brh-0 in shoe games, where Brh-0 outperforms Hi-Lo for play all, but not for some of the Wonging cases.

Using the table above, if the player wishes to bet $5 to $40, the unit bet is $5. The optimal scheme is obtained by multiplying each value in the column by $5$, and rounding to the nearest bettable unit. This results in a maximum bet of $40$ at TC=+6. Now, $ekb$ is equal to 633 units here, so that the $EKB$ for the $5$-$40$ betting scheme is $3165$. For a fixed bettor, this is the amount for which there is a 13.6% risk of ruin, generally felt to be unacceptably high. The risk of ruin can be lowered to whatever is desired by having a larger bankroll. How much larger can be seen in the following table:

<table>
<thead>
<tr>
<th>ROR</th>
<th>x EKB</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.6%</td>
<td>1.0</td>
</tr>
<tr>
<td>5.0%</td>
<td>1.5</td>
</tr>
<tr>
<td>2.0%</td>
<td>2.0</td>
</tr>
<tr>
<td>1.0%</td>
<td>2.3</td>
</tr>
<tr>
<td>0.1%</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Therefore, to have a $ROR$ of 5%, the player would require a bankroll of $1.5 \times 3165$, which is $4747$. This would be rounded in practice to $4750$. The ROR is equal to

$$ ROR = e^{-2 (Bankroll / EKB)} $$

where $e=2.718$ is the Natural number. The derivation of this result is given in Appendix B of the Book, and was originally derived by Patrick Sileo.

Alternatively, if a player has a $10000$ bankroll, and desires a 2% $ROR$, then the $EKB$ must be $10000/2.0=5000$. But if the $EKB$ is $5000$, then the betting unit $SB$ must be $EKB/ekb=5000/633=7.90$. Now while it is impossible to actually bet $7.90$, the minimum bet will be $5$ or maybe occasionally $10$ for cover purposes, and each intermediate bet will be $7.90$ times each unit value up to a maximum bet of $60$.

Unfortunately, there are table minimums, so that for a given game there will always be a minimum value of $SB$. This means that while choosing an optimal spread, there will either be a minimum bankroll.
requirement or a commitment to top up later if the funds are not immediately available. The play-all spreads are a prime example of this.

**Calculation of Win-Rate:**

For the all important win rate, $N_0$ re-enters the picture. Using Equations (1)-(3) we get

$$EV = \frac{B \cdot ekb}{N_0} = \frac{EKB}{N_0} ,$$

which leads to another interpretation of $N_0$:

*The expectation per hand is $1/N_0$ times the Equivalent Kelly Bankroll.*

So for the $5-$40, play only $UTC \geq -6$ scheme used above, the expected win is $EV = \frac{3165}{31208} = \$0.1014$ per round, or $\$10.14$ per 100 rounds.

Don Schlesinger has promoted a measure called 'SCORE', which is the $EV$ obtained from an optimal betting scheme using an $EKB$ of $\$10000$, per 100 rounds. This means that a fixed bettor with a $\$10000$ $EKB$ will win $SCORE$ dollars per 100 rounds. Alternatively, a player needs a $\$15000$ bankroll in order to have a 5% $ROR$ with a win rate given by $SCORE$. I personally feel that $N_0$ conveys more information, because $SCORE$ is only a win rate, not immediately conveying any sense of variance to those unfamiliar with optimal spread theory. Schlesinger believes that players relate more to win rates rather than time frames, I would certainly acknowledge that he has had vastly more experience in this area than myself. If you have read the $SCORE$ article in 'Blackjack Forum' and relate to $SCORE$ rather than $N_0$, then you can convert $N_0$ to $SCORE$ as follows:

$$SCORE = \left(\frac{\$10000}{N_0}\right) \times 100 .$$

A note now on spreading to two hands. The $ROR$ will increase for a fixed bettor, if a $5-$40 single-hand spread, is changed to a $5-$40 two-hand spread. This is because the maximum bet of $\$40$ is played on BOTH hands, making a total of $\$80$. The increase in the $ekb$ value in the tables illustrates this fact. Whether this is a 1-8 or 1-16 spread is a question of interpretation. Generally, provided the max bet is reduced by 80%, then the $ROR$ will be maintained. So if you are alternating in real play between a single-hand and two-hands, you will need to switch between say a 1-10 single-hand spread and a 1-8 two-hand spread, just make sure the $ekb$ values are similar. Playing more than two hands uses up too many cards in the positive counts, without enough gain from the extra hands if you are betting correctly.

**Proportional or Kelly Bettors**

A proportional bettor will choose the value of $B$ given the current value of the bankroll, and the Kelly factor $k$.

$$B = \frac{k \cdot Bankroll}{ekb} ,$$

(9)
and so

\[ EKB = k \cdot \text{Bankroll} \]  

(10)

where \( ekb \) is the Unit EKB for the betting spread. So for the 1-8 spread above, with \( ekb=633, k=1 \), and a bankroll of $10000, the value of $B$ is $15.80. So the minimum bet will be $15 and the maximum bet $125, with the intermediate bets equal to $15.80 times the values in the table, rounded down to the nearest bettable amount. If instead we had chosen \( k=0.8 \) here, the unit bet would be $12 instead.

As the player wins or loses, the unit bet $B$ is reassessed. If possible, resizing should occur after no more than a 10% change in bankroll. Betting in this way is expected to double the Bankroll every

\[ \tau = \frac{0.693 \cdot N0}{k - k^2/2} \text{ rounds.} \]  

(11)

This compares to a fixed bettor who is expected to win an amount equal to the EKB, every \( N0 \) hands. The difference is that this is a linear win rate, whereas the Kelly bettor will keep doubling, and the bankroll will, in theory at least, rise exponentially. Note that the doubling time is maximised for \( k=1 \), where the doubling time becomes 1.39 \( N0 \).

For the optimal spread above, we have \( ekb=633, k=1 \), so that for a $5000 bankroll, we have a unit bet B=$7.90. This value applied to the spread gives the second column, which is the closest spread with bettable units. Note that the spread is no longer 1-8, it is now 1-12. This is an unfortunate consequence of real world bet sizing.

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<td>$60</td>
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\[ ekb \quad 633 \quad \$5000 \]

\[ N0 \quad 31208 \]

\[ (\text{Hi-Lo}) \quad 35678 \]

\[ \%\text{Pl} \quad 72.66 \]

A player now faces some choices. If the player were able to play 1-12, then it would be better to actually use a 1-12 optimal spread. Another possibility is to mix up the $5 bets with some $10 bets, to make the spread look a little less aggressive.